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ATOMIC THEORIES OF ENERGY.

A THEORY involving some sort of a discrete or discontinuous structure of energy has been put forward by Prof. Max Planck of the University of Berlin. The various aspects of this theory are discussed and elaborated by the late M. Henri Poincaré in a paper entitled "L'Hypothèse des Quanta," published in the *Revue Scientifique* (Paris, Feb. 21, 1912).

A paper in which a discontinuous or "atomic" structure of energy was suggested was prepared by the present writer fifteen years ago but remains unpublished for reasons that will appear later. Although he has no desire to put in a claim of priority and is well aware that failure to publish would put any such claim out of court, it seems to him that in connection with present radical developments in physical theory the paper, together with some correspondence relating thereto, has historical interest. Planck's theory was suggested by thermodynamical considerations. In the paper now to be quoted the matter was approached from the standpoint of a criterion for determining the identity of two portions of matter or of energy. The paper is as follows:

SOME CONSIDERATIONS ON THE IDENTITY OF DEFINITE PORTIONS OF ENERGY.

It has been remarked recently that physicists are now divided into two opposing schools according to the way in which they view the subject of energy, some regarding it as a mere mathematical abstraction and others looking upon it as a physical entity, filling space and continuously migrating by definite paths from one place to another. It may be added that there are numerous factions within these two parties; for instance, not all of those who consider energy to be something more than a mere mathematical expression would maintain that a given quantity of it retains its identity just as a given quantity of matter does. In fact a close analysis would possibly show that opinions are graded very closely and continuously from a view hardly differing from that of Lagrange, who clearly saw and freely used the mathematical considerations involving energy before the word had been invented or its physical meaning developed, up to that stated recently in its extreme form by Professor Ostwald, who would replace what he terms a mechanical theory of the universe by an "energetical" theory, and would dwell exclusively on energy as opposed to its vehicles.

Differences of opinion of this sort very frequently reduce to differences of definition, and in this case the meaning of the word "identity" or some similar word or phrase has undoubtedly much to do with the view that is taken of the matter. It may be interesting, for instance, to look for a moment at our ideas of the identity of matter and the extent to which they are influenced by the accepted theory of its constitution.

Very few persons would hesitate to admit that the matter that now constitutes the universe is identical in amount with that which constituted it one million years ago, and that any given portion of that matter is identical with an equal amount of matter that then existed, although the situations of the parts of that portion might be and probably were widely different in the two cases. To assert this is of course a very different thing from asserting that the identity of the two portions or any parts thereof could have been practically shown by following them during all their changes of location or state. That cannot be done even in the case of some simple changes that are effected in a fraction of a second. For instance, if water from the pail A be mixed with water from the pail B there is no possible way of telling which pail any given portion of the mixture came from or in what proportions, yet it is certain that such portion is identical with a portion of equal mass that recently occupied part of one or both pails.

How far our certainty as to this is influenced by our ideas regarding the ultimate constitution of the water is worthy of investigation. All who accept the molecular theory, for instance, will regard our inability to trace the elements of a mixture as due to purely physical limitations. A set of Maxwell's "demons" if bidden to watch the molecules of the water in pail A, one demon being assigned to each molecule, would be able to tell us at any time the precise proportions of any given part of the mixture. But if we should not accept the molecular theory and believe for instance, that water is a continuum, absolutely homogeneous, no matter how small portions of it be selected, then our demons would be as powerless as we ourselves now are to trace the constituents in the mixture.

We are now in a position to ask the question: Is the matter in a mixture of two continua identical with that of its constituents? The identity certainly seems of a different kind or degree from that which obtains in the first case, for there is no part, however small, that was derived from one pail alone. The mixture is something more than a mere juxtaposition of elements each of which has retained its identity; it is now of such nature that no part of it is identical with any part of A alone or of B alone, nor of A+B, where the sign + denotes simple juxtaposition. It is identical, to be sure, with a perfect mixture of certain parts of A and B, but this is simply saying that it is identical with what it is now, that is, with itself, not with something that went before.

Probably no one now believes that water or any other kind of matter is a continuum, but the bearing of what has been said may be seen when we remember that this is precisely the present stage of our belief regarding energy.

No one, so far as I know, has ventured to suggest what may be termed a molecular theory of energy, a somewhat remarkable fact when we consider the control now exercised over all thought in physics by molecular theories of matter. While we now believe, for instance, that a material body, say a crystal, can by no possibility increase continuously in mass, but must do so step by step, the minimum mass of matter that can be added being the molecule, we believe on the contrary that the energy possessed by the same body can and may increase with absolutely perfect continuity, being hampered by no such restriction.

It is not the purpose of this paper to discuss whether we have grounds for belief that there is such a thing as a minimum quantity, or atom, of energy, that does not separate into smaller parts, no matter what changes it undergoes. Suffice it to say that there appears to be no *a priori* absurdity in such an idea. At first sight both matter and energy appear non-molecular in structure. But we have been forced to look upon the gradual growth of a crystal as a step-by-step process, and we may some day, by equally cogent considerations, be forced to regard the gradual increase of energy of an accelerating body as also a step-by-step process, although the discontinuity is as invisible to the eye in the latter case as in the former.

Without following this out any farther, however, the point may be here emphasized that it is hardly possible for one who, like the majority of physicists, regards matter as molecular and energy as a continuum, to hold the same ideas regarding the identity of the two. Efforts to show that definite portions of energy, like definite portions of matter, retain their identity have hitherto been made chiefly on the lines of a demonstration that energy travels by definite and continuous paths in space just as matter does. This is very well, but it would appear to be necessary to supplement it with evidence to show that the lines representing these paths do not form at their intersections continuous blurs that not only forbid any practical attempt at identification on emergence, but make it doubtful whether we can in any true sense call the issuing path identical with the entering one. Otherwise the identity of energy can be admitted to be only that kind of identity that could be preserved by matter if its molecular structure did not exist. One who can admit that this sort of identity is the same sort that can be preserved by molecular matter may be able to hold the identity of energy in the present state of the evidence, but the present attitude of physicists would seem to show that, whether they realize the connection of the two subjects or not, they cannot take this view. In other words, modern views of the identity of matter seem closely connected with modern views of its structure, and the same connection will doubtless hold good for energy.

Regarding the probable success of an attempt to prove that energy has a "structure" analogous to the molecular structure of matter, any prediction would doubtless be rash just now. The writer has been unable, up to the present time, to disprove the proposition, but the subject is one of corresponding importance to that of the whole molecular theory of matter and should not be entered upon lightly.

* * *

The writer freely acknowledges at present that the illustrations in the foregoing are badly chosen and some of the statements are too strong, but it still represents essentially his ideas on the subject. No reputable scientific journal would undertake to publish it. The paper was then sent to Prof. J. Willard Gibbs of Yale, and elicited the following letter from him:

"New Haven, June 2/97.

"My Dear Mr. Bostwick:

"I regret that I have allowed your letter to lie so long unanswered. It was in fact not very easy to answer, and when one lays a letter aside to answer, the weeks slip away very fast.

"I do not think that you state the matter quite right in regard to the mixture of fluids if they were continuous. The mixing of water as I regard it would be like this, if it were continuous and not molecular. Suppose you should take strips of white and red glass and heat them until soft and twist them together. Keep on drawing them out and doubling them up and twisting them together. It would soon require a microscope to distinguish the red and white glass, which would be drawn out into thinner and thinner filaments if the matter were continuous. But it would be always only a matter of optical power to distinguish perfectly the portions of red and white glass. The stirring up of water from two pails would not really mix them but only entangle filaments from the pails.

"To come to the case of energy. All our ideas concerning energy seem to require that it is capable of gradual increase. Thus the energy due to velocity can increase continuously if velocity can. Since the energy is as the square of the velocity, if the velocity can only increase discontinuously by equal increments, the energy of a body will increase by unequal increments in such a way as to make the exchange of energy between bodies a very awkward matter to adjust.

"But apart from the question of the increase of energy by discontinuous increments, the question of relative and absolute motion makes it very hard to give a particular position to energy. Since the 'energy' we speak of in any case is not one quantity but may be interpreted in a great many ways. Take the important case of two equal elastic balls. One, moving, strikes the other at rest, we say, and gives it nearly all its energy. But we have no right to call one ball at rest and we can not say (as anything absolute) which of the balls has lost and which has gained energy. If there is such a thing as absolute energy of motion it is something entirely unknowable by us. Take the solar system, supposed isolated. We may take as our origin of coordinates the center of gravity of the system. Or we may take an origin with respect to which the center of gravity of the solar system has any (constant) velocity. The kinetic energy of the earth, for example, may have any value whatever, and the principle of the conservation of energy will hold in any case for the whole solar system. But the shifting of energy from one planet to another will take place entirely differently when we estimate the energies with reference to different origins.

"It does not seem to me that your ideas fit in with what we know about nature. If you ask my advice, I should not advise you to try to publish them.

"At best you would be entering into a discussion (perhaps not in bad company) in which words would play a greater part than precise ideas.

"This is the way I feel about it.

"I remain
"Yours faithfully,
J. W. GIBBS."

Professor Gibbs's criticism of the illustration of water-mixture is evidently just. Another might well have been used where the things mixed are not material—for instance the value of money deposited in a bank. If A and B each deposits $100 to C's credit and C then draws $10, there is evidently no way of determining what part of it came from A and what from B. The structure of "value," in other words, is perfectly continuous. Professor Gibbs's objections to an "atomic" theory of the structure of energy are most interesting. The difficulties that it involves are not overstated. In 1897 they made it unnecessary, but since that time considerations have been brought forward, and generally recognized, which may make it necessary to brave those difficulties.

Planck's theory was suggested by the apparent necessity of modifying the generally accepted theory of statistical equilibrium involving the so-called "law of equipartition," enunciated first for gases and extended to liquids and solids.

In the first place the kinetic theory fixes the number of degrees of freedom of each gaseous molecule, which would be three for argon, for instance, and five for oxygen. But what prevents either from having the six degrees to which ordinary mechanical theory entitles it? Furthermore, the oxygen spectrum has more than five lines, and the molecule must therefore vibrate in more than five modes. "Why," asks Poincaré, "do certain degrees of freedom appear to play no part here; why are they, so to speak, 'ankylosed'?" Again, suppose a system in statistical equilibrium, each part gaining on an average, in a short time, exactly as much as it loses. If the system consists of molecules and ether, as the former have a finite number of degrees of freedom and the latter an infinite number, the unmodified law of equipartition would require that the ether should finally appropriate all energy, leaving none of it to the matter. To escape this conclusion we have Rayleigh's law that the radiated energy, for a given wavelength, is proportional to the absolute temperature, and for a given temperature is in inverse ratio to the fourth power of the wave-length. This is found by Planck to be experimentally unverifiable, the radiation being less for small wave-lengths and low temperatures, than the law requires.

Still again, the specific heats of solids, instead of being sensibly constant at all temperatures, are found to diminish rapidly in the low temperatures now available in liquid air or hydrogen and apparently tend to disappear at absolute zero. "All takes place," says Poincaré, "as if these molecules lost some of their degrees of freedom in cooling —as if some of their articulations froze at the limit."

Plank attempts to explain these facts by introducing the idea of what he calls "quanta" of energy. To quote from Poincaré's paper:

"How should we picture a radiating body? We know that a Hertz resonator sends into the ether Hertzian waves that are identical with luminous waves: an incandescent body must then be regarded as containing a very great number of tiny resonators. When the body is heated, these resonators acquire energy, start vibrating and consequently radiate.

"Planck's hypothesis consists in the supposition that each of these resonators can acquire or lose energy only by abrupt jumps, in such a way that the store of energy that it possesses must always be a multiple of a constant quantity, which he calls a 'quantum'—must be composed of a whole number of quanta. This indivisible unit, this quantum, is not the same for all resonators; it is in inverse ratio to the wave-length, so that resonators of short period can take in energy only in large pieces, while those of long period can absorb or give it out by small bits. What is the result? Great effort is necessary to agitate a short-period resonator, since this requires at least a quantity of energy equal to its quantum, which is great. The chances are, then, that these resonators will keep quiet, especially if the temperature is low, and it is for this reason that there is relatively little short-wave radiation in 'black radiation.'... The diminution of specific-heats is explained similarly: When the temperature falls, a large number of vibrators fall below their quantum and cease to vibrate, so that the total energy diminishes faster than the old theories require."

Here we have the germs of an atomic theory of energy. As Poincaré now points out, the trouble is that the quanta are not constant. In his study of the matter he notes that the work of Prof. Wilhelm Wien, of Würzburg, leads by theory to precisely the conclusion announced by Planck that if we are to hold to the accepted ideas of statistical equilibrium the energy can vary only by quanta inversely proportional to wave-length. The mechanical property of the resonators imagined by Planck is therefore precisely that which Wien's theory requires. If we are to suppose atoms of energy, therefore, they must be variable atoms. There are other objections which need not be touched upon here, the whole theory being in a very early stage. To quote Poincaré again:

"The new conception is seductive from a certain standpoint: for some time the tendency has been toward atomism. Matter appears to us as formed of indivisible atoms; electricity is no longer continuous, not infinitely divisible, it resolves itself into equally-charged electrons; we have also now the magneton, or atom of magnetism. From this point of view the quanta appear as *atoms* of *energy*. Unfortunately the comparison may not be pushed to the limit; a hydrogen atom is really invariable.... The electrons preserve their individuality amid the most divers vicissitudes, is it the same with the atoms of energy? We have, for instance, three quanta of energy in a resonator whose wave-length is 3; this passes to a second resonator whose wave-length is 5; it now represents not 3 but 5 quanta, since the quantum of the new resonator is smaller and in the transformation the number of atoms and the size of each has changed."

If, however, we replace the atom of energy by an "atom of action," these atoms may be considered equal and invariable. The whole study of thermodynamic equilibrium has been reduced by the French mathematical school to a question of probability. "The probability of a continuous variable is obtained by considering elementary independent domains of equal probability.... In the classic dynamics we use, to find these elementary domains, the theorem that two physical states of which one is the necessary effect of the other are equally probable. In a physical system if we represent by q one of the generalized coordinates and by p the corresponding momentum, according to Liouville's theorem the domain $\int\int dpdq$, considered at a given instant, is invariable with respect to the time if p and q vary according to Hamilton's equations. On the other hand p and q may, at a given instant take all possible values, independent of each other. Whence it follows that the elementary domain is infinitely small, of the magnitude $dpdq$.... The new hypothesis has for its object to restrict the variability of p and q so that these variables will only change by jumps....Thus the number of elementary domains of probability is reduced and the extent of each is augmented. The hypothesis of quanta of action consists in supposing that these domains are all equal and no longer infinitely small but finite and that for each $\int\int dp\, dq = h$, h being a constant."

Put a little less mathematically, this simply means that as energy equals action multiplied by frequency, the fact that the quantum of energy is proportional to the frequency (or inversely to the wave-length as stated above) is due simply to the fact that the quantum of action is constant—a real atom. The general effect on our physical conceptions, however, is the same: we have a purely discontinuous universe—discontinuous not only in matter but in energy and the flow of time. M. Poincaré thus puts it:

"A physical system is susceptible only of a finite number of distinct states; it leaps from one of these to the next without passing through any continuous series of intermediate states."

He notes later:

"The universe, then, leaps suddenly from one state to another; but in the interval it must remain immovable, and the divers instants during which it keeps in the same state can no longer be discriminated from one another; we thus reach a conception of the discontinuous variation of time —the *atom of time*."

I quote in conclusion, Poincaré's final remarks:

"The present state of the question is thus as follows: the old theories, which hitherto seemed to account for all the known phenomena, have met with an unexpected obstacle. Seemingly a modification becomes necessary. A hypothesis has presented itself to M. Planck's mind, but so strange a one that one is tempted to seek every means of escaping it; these means, however, have been sought vainly. The new theory, however, raises a host of difficulties, many of which are real and not simply illusions due to the indolence of our minds, unwilling to change their modes of thought.....

"Is discontinuity to reign throughout the physical universe, and is its triumph definitive? Or rather shall we find that it is but apparent and hides a series of continuous processes?....To try to give an opinion just now on these questions would only be to waste ink."

It only remains to call attention again to the fact that this conception of the discontinuity of energy, the acceptance of which Poincaré says would be "the most profound revolution that natural philosophy has undergone since

Newton" was suggested by the present writer fifteen years ago. Its reception and serious consideration by one of the first mathematical physicists of the world seems a sufficient justification of its suggestion then as a legitimate scientific hypothesis.

Arthur E. Bostwick.

St. Louis, Mo.